Ancient Declinations and Precession

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In *Almagest* 7.3 Ptolemy lists the declinations of 18 stars from the time of Timocharis and Aristyllos, from the time of Hipparchus, and from his own time.[1] For six of the stars he says that the change in declination over the period of 265 years between his time and Hipparchus' time corresponds closely to the change in declination of the endpoints of various segments of the ecliptic that are 2 2/3° in length. Ptolemy uses these correspondences to claim that the sphere of the fixed stars is rotating eastward about the poles of the ecliptic 1° every 100 years, in agreement with several alternative determinations of the rate of precession that he offers nearby in the *Almagest* (and, of course, in disagreement with the correct value of 72 years per degree).

However, Ptolemy gave the positions of these segments only roughly, within signs of the zodiac, e.g. "near the middle of Taurus." If the positions are ecliptic longitudes, as most previous commentators have assumed,[2] then some are grossly inaccurate. For example, for η Ursae Majoris he puts the segment near the beginning of Libra, or 180°, while the longitude of the star at his time was actually close to the beginning of Virgo, or 150°. Manitius assumed that they were polar longitudes,[3] while Rawlins more recently speculated that they might refer to right ascensions.[4] Polar longitudes and right ascensions cannot be distinguished conclusively in this case, since Ptolemy tells us only roughly where the segments lie – at the end of Aries, near the middle of Taurus, etc. However, the fact that Ptolemy did not mention the ecliptical longitudes of the stars in question suggests that he probably did *not* analyze the changes in declination in the same way that modern commentators have, and raises the question of exactly *how* he did analyze them. The following discussion suggests one approach, admittedly speculative, to answering that question.

Let us begin by thinking about the situation as it might have appeared, not to Ptolemy, but to Hipparchus. Let us assume that Hipparchus had that list of 18 declinations from Timocharis and Aristyllos,[5] although he might have been unsure of their dates, and particularly the distinction between the dates of Timocharis (*ca.* –290) and Aristyllos (*ca.* –260)[6]. He was certainly able to measure the declinations of those same stars in his own time, since the declinations of several of them and many others appear in his *Commentary to Aratus*. Therefore let us suppose that Hipparchus knew that the declinations were changing with time, and that he wondered why. Following additional hints left us by Ptolemy, let us further suppose that he formulated the hypothesis that the sphere of the fixed stars was rotating about the pole of the ecliptic, and he needed to use the changes in declination to estimate how fast. How would he do that?

Hipparchus did not have the earlier right ascensions, so he could not simply calculate the earlier longitudes and thereby the change in longitude over the intervening years.[7] However, he might have settled upon the following alternative algorithm, which follows in style the calculations he tells us about directly in his *Commentary*:

(1) assume that if we are given a value α of right ascension, then we can calculate the point π on the ecliptic that has the same right ascension, and the declination μ of that point.

(2) use the known right ascension α_2 of the star in his time to compute the corresponding point π_2 on the ecliptic.

(3) compute the declination μ_2 of the point at π_2 on the ecliptic.

(4) *assume* that the change in declination of the interval on the ecliptic is the same as the known change in declination of the star, $\Delta\delta = \delta_2 - \delta_1$, and so compute the declination μ_1 of the earlier point π_1 using $\mu_1 = \mu_2 - \Delta\delta$.

(5) compute the point π_1 on the ecliptic that has declination μ_1.

(6) finally, *assume* that the change in polar longitude, $\pi_2 - \pi_1$, is a good approximation to the actual change in longitude of the star in question.

Of course, this algorithm is just one way to explain the rather terse discussion that Ptolemy gives for each pair of declinations. We do know from an explicit example in the *Commentary* that Hipparchus was familiar with similar sequences of calculations, and that he routinely computed the right ascension and declination of any point on the ecliptic, the only non-trivial steps in the algorithm, but we cannot be sure whether his computations used trigonometry (the relevant formulae are $\tan\pi = \tan\alpha / \cos\varepsilon$, where ε is the obliquity of the ecliptic, and $\sin\mu = \sin\varepsilon \sin\pi$) or an analog method such as a globe.[8] Either way, though, Hipparchus would likely know that the algorithm gives a good approximation to the change in longitude of the star, and that it was not exact. He would likely also know that the result of the algorithm is insensitive to moderate variations in the assumed input right ascensions.

When the algorithm is applied to the 18 stars in the list in *Almagest* 7.3, we get the results shown in Table 1 and Figure 1. The two stars which show no change in declination (because they are so near the solstitial points) yield, of course, no result and are omitted from Fig. 1. They might, though, lead Hipparchus to exclude from further consideration all the stars for which the change in declination is relatively small.

What might Hipparchus conclude from these results? As far as we know he had no concept of averaging, or even how to draw a chart like Fig. 1 to get a visual impression of the data. However, Hipparchus probably knew that his algorithm was yielding only an approximation to the precession constant, and he might also have been uncertain about the length of time that separated him from Timocharis' and Aristyllos' declinations (as indeed we are today; for simplicity, I have used a uniform 165 years). So under the circumstances, the conclusion that Ptolemy *reports* that Hipparchus drew as a summary of all his investigations of precession, that the change in longitude is *at least* 1° per 100 years, appears to me eminently reasonable. And given the results shown, Hipparchus can certainly be excused for not finding the *correct* value of 1° per 72 years.

It is also possible that for reasons now lost Hipparchus decided to base his estimate on the same six stars that Ptolemy chooses in *Almagest* 7.3 (a roundabout way of saying that perhaps Ptolemy chose those six simply because Hipparchus had chosen them). Perhaps the distribution in declination of those six stars appealed to Hipparchus in his effort to resolve whether precession was a property only of stars near the ecliptic or included all stars. In any event, for those six stars it happens that the average estimate of the precession constant is indeed about 98 years/degree, so it would be all the more understandable how Hipparchus got his conclusion. To be fair, though, we must also mention that if Hipparchus used a shorter time interval than what I have assumed, then his estimated value in years per degree would be correspondingly smaller, and hence closer to the correct value. But we have no information about what dates Hipparchus might have been using.

Now we move forward to Ptolemy. If the scenario sketched above is anywhere near what actually happened, then it is likely, as first suggested by Rawlins[9], that Ptolemy read about it in one of Hipparchus' now-lost books and is, in *Almagest* 7.3, simply echoing it, either in a form similar to what Hipparchus wrote, or in summary form. Indeed, applying the algorithm to the declination changes between Ptolemy's time and Hipparchus' time yields the results in Table 2 and Figure 2. For the six stars that Ptolemy singles out for analysis the algorithm yields 93 years/degree. Three of the stars yield values just under 100 years/degree, certainly close enough that Ptolemy is justified in calling them 'the same' as his expected result,[10] while the other three yield somewhat smaller values, and for those three Ptolemy in fact does write that the agreement is only 'near' or 'approximate'. Thus it is quite possible that Ptolemy was simply rounding his six values, and also telling us about the three cases he was rounding the most.[11]

Neugebauer[12] and Toomer[13] have argued that the mere existence of the discussion of declinations in the *Almagest* shows that Ptolemy could not have inherited a table of stellar ecliptical coordinates from Hipparchus, otherwise why would Ptolemy have resorted to such a 'cumbersome process of comparing declinations'. One way to answer this argument is to agree with it, and assume that Ptolemy did *not* inherit a table of ecliptical coordinates, but rather a table of equatorial coordinates. Let us see what evidence we can present from Ptolemy himself to support this scenario.

First, the discussion above of the *Almagest* declination passages suggests that Ptolemy was simply duplicating and updating the same analysis that Hipparchus had published some 265 years earlier, and as we have seen there is no appearance whatsoever of ecliptical coordinates in that analysis, just as ecliptical coordinates play no role in Hipparchus' *Commentary*. Indeed, the evidence in Hipparchus' *Commentary to Aratus*[14] suggests that Hipparchus worked routinely in right ascension and declination,[15] and not, as often supposed, in some form of mixed coordinates.[16] Thus the above discussion is consistent with the idea that Hipparchus was working in equatorial coordinates.

Second, Ptolemy writes in the final words of *Almagest* 7.2 "…their individual distances [in ecliptic longitude] from the solstitial or equinoctial points are in each case about 2 2/3° farther to the rear than those *derivable from what Hipparchus recorded* [italics added]."[17] Hence it seems that Ptolemy is well acquainted with the idea of *deriving* ecliptical results from Hipparchus' data, a required process under the proposed scenario.

Third, Ptolemy writes in *Almagest* 7.3 "…we find that it [ecliptical longitude] is practically the same as that *computed from the records of Hipparchus.*"[18] Although Ptolemy doesn't explicitly say that *he* is doing the computing, he is clearly saying that some computation has been done, presumably because it was necessary.

Fourth, in *Almagest* 7.4 Ptolemy goes to some pains to explain to us that his use of 'to the rear of' and 'in advance of' and 'to the north of' and 'to the south of' refer directly to ecliptical coordinates. However, there are several cases where his star descriptions use this terminology but are not in accord with the facts. Toomer points out several examples of this.[19] It is interesting, though, that in each case the wording *is accurate* in equatorial coordinates. So it is plausible that Ptolemy copied the star descriptions he used from some Hipparchan document that was accurate in equatorial coordinates, but occasionally forgot to change them to be uniformly accurate not for equatorial coordinates but for ecliptical coordinates. It is also possible that someone else, perhaps even Hipparchus himself, did the conversion and forgot to change some of the descriptions, but it seems most likely that the person farthest from the original data is the person most likely to make such oversights.

Finally, regarding potential star identification problems in his catalogue, Ptolemy writes in *Almagest* 7.4 "one has a ready means of identifying those stars which are described differently [by others]; this can be done immediately simply by comparing the recorded positions." This passage clearly implies that Ptolemy was not the first to use ecliptical coordinates in a star catalogue, and further, since he says the comparison may be done 'immediately', Ptolemy is probably also telling us that other star catalogues in ecliptical coordinates were readily available.[20] So by providing his new table in ecliptical coordinates, Ptolemy is presumably simply conforming to the standard presentation of his day. There is no corresponding evidence that Hipparchus ever felt such a motivation in his day, speculations about his discovery of precession notwithstanding.

Besides these passages from Ptolemy himself, it is worthwhile to recall also the historical summary of Dreyer, that "precession is never alluded to by Geminus, Kleomedes, Theon of Smyrna, Manilius, Pliny, Censorinus, Achilles, Chalcidius, Macrobius, Martianus Capella!"[21] Although 'absence of evidence is not evidence of absence', surely these omissions, coupled with Ptolemy's statement that at most Hipparchus attached only a lower bound on the rate of precession, suggest that Hipparchus himself never reached a firm and final conclusion on the phenomenon, and so might well have been content to remain in equatorial coordinates.

Together, then, these passages suggest that Ptolemy himself *might* have derived the longitudes from an Hipparchan table of equatorial coordinates. While far from conclusive, the argument at least has multiple instances of textual support. Therefore, it would be interesting to find additional evidence that either favors or disfavors this scenario, perhaps in the data of the *Almagest* star catalog itself.

Acknowledgment

I thank Dennis Rawlins for bringing his paper in ref. 2 to my attention, Alexander Jones for a discussion of the translation issue mentioned in footnote 18, and Hugh Thurston for a number of helpful suggestions and comments.

Name		Δδ	α_2	π_2	μ_2	μ_1	π_1	$\pi_2 - \pi_1$	Δt	p	
α Aql		0.00	271.6	271.5	-23.9	-23.9	271.5	0.00	165	NA	
η Tau	*	0.67	26.8	28.9	11.3	10.6	27.1	1.83	165	90.0	*
α Tau		1.00	39.5	42.0	15.7	14.7	38.9	3.14	165	52.6	
α Aur	*	0.40	42.1	44.7	16.5	16.1	43.3	1.32	165	125.1	*
γ Ori	*	0.60	53.3	55.7	19.5	18.9	53.3	2.41	165	68.3	*
α Ori		0.50	60.4	62.6	21.0	20.5	60.2	2.41	165	68.4	
α CMa		0.33	77.9	78.9	23.4	23.1	75.5	3.41	165	48.4	
α Gem		0.17	79.1	80.0	23.5	23.3	78.0	1.98	165	83.4	
β Gem		0.00	83.1	83.7	23.7	23.7	83.7	0.00	165	NA	
α Leo		-0.67	122.7	120.5	20.4	21.1	117.3	3.19	165	51.7	
α Vir	*	-0.80	174.0	173.4	2.7	3.5	171.5	1.99	165	82.8	*
η UMa	*	-0.75	184.6	185.1	-2.0	-1.3	183.2	1.86	165	88.8	*
ζ UMa		-0.75	177.3	177.0	1.2	2.0	175.1	1.86	165	88.8	
ε UMa		-0.90	166.4	165.2	5.9	6.8	162.9	2.30	165	71.7	
α Boo	*	-0.50	189.7	190.5	-4.2	-3.7	189.3	1.25	165	131.8	*
α Lib		-0.60	194.5	195.8	-6.3	-5.7	194.3	1.53	165	108.0	
β Lib		-0.80	201.6	203.4	-9.2	-8.4	201.3	2.11	165	78.1	
α Sco		-0.67	216.2	218.7	-14.7	-14.0	216.7	2.02	165	81.7	

Table 1. Results for the estimate of precession using the change in declination between the time of Timocharis and Aristyllos (assumed as –293) and Hipparchus (assumed as –128). The rows marked with * are the six stars Ptolemy analyzes in *Almagest* 7.3. All angles are in degrees. The estimated precession constant *p* is given in years per degree.

Name		$\Delta\delta$	α_2	π_2	μ_2	μ_1	π_1	$\pi_2 - \pi_1$	Δt	p	
α Aql		0.03	274.9	274.5	-23.8	-23.8	273.4	1.11	265	239.6	
η Tau	*	1.08	30.4	32.6	12.6	11.5	29.6	3.06	265	86.6	*
α Tau		1.25	43.0	45.6	16.8	15.5	41.5	4.10	265	64.7	
α Aur	*	0.77	46.4	48.9	17.7	17.0	46.2	2.68	265	98.8	*
γ Ori	*	0.70	56.7	59.0	20.3	19.6	56.0	3.03	265	87.5	*
α Ori		0.92	63.9	65.9	21.7	20.7	61.1	4.73	265	56.0	
α CMa		0.25	80.8	81.5	23.6	23.3	78.3	3.24	265	81.8	
α Gem		0.23	83.4	83.9	23.7	23.5	80.1	3.81	265	69.5	
β Gem		0.17	87.2	87.5	23.8	23.7	83.0	4.52	265	58.6	
α Leo		-0.83	126.5	124.1	19.6	20.4	120.5	3.62	265	73.1	
α Vir	*	-1.10	177.3	177.1	1.2	2.3	174.4	2.73	265	97.2	*
η UMa	*	-1.08	187.7	188.4	-3.4	-2.3	185.8	2.70	265	98.3	*
ζ UMa		-1.50	180.8	180.8	-0.3	1.2	177.1	3.71	265	71.4	
ε UMa		-1.35	170.6	169.7	4.1	5.5	166.3	3.40	265	77.9	
α Boo	*	-1.17	192.7	193.9	-5.6	-4.4	191.0	2.94	265	90.1	*
α Lib		-1.57	197.9	199.4	-7.7	-6.2	195.4	4.03	265	65.7	
β Lib		-1.40	204.9	206.9	-10.6	-9.2	203.2	3.77	265	70.4	
α Sco		-1.25	219.9	222.4	-15.8	-14.6	218.5	3.92	265	67.6	

Table 2. Results for the estimate of precession using the change in declination between the time of Hipparchus (assumed as –128) and Ptolemy (assumed as +137). The rows marked with * are the six stars Ptolemy analyzes in *Almagest* 7.3. All angles are in degrees. The estimated precession constant *p* is given in years per degree.

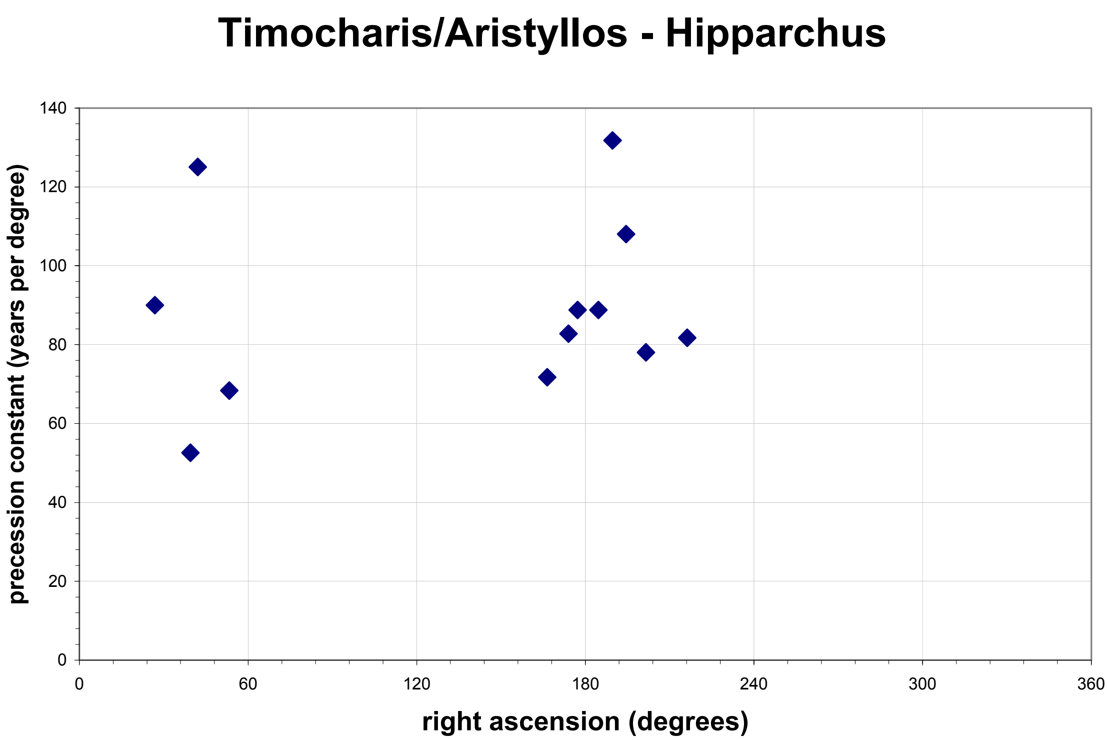

Figure 1. The precession constants for Hipparchus' stars. Four stars (α Ori, α Cma, α Gem, and α Leo) are so near a solstice that Hipparchus might have decided to exclude them from consideration. The average value for the 12 stars shown is 89 years/degree.

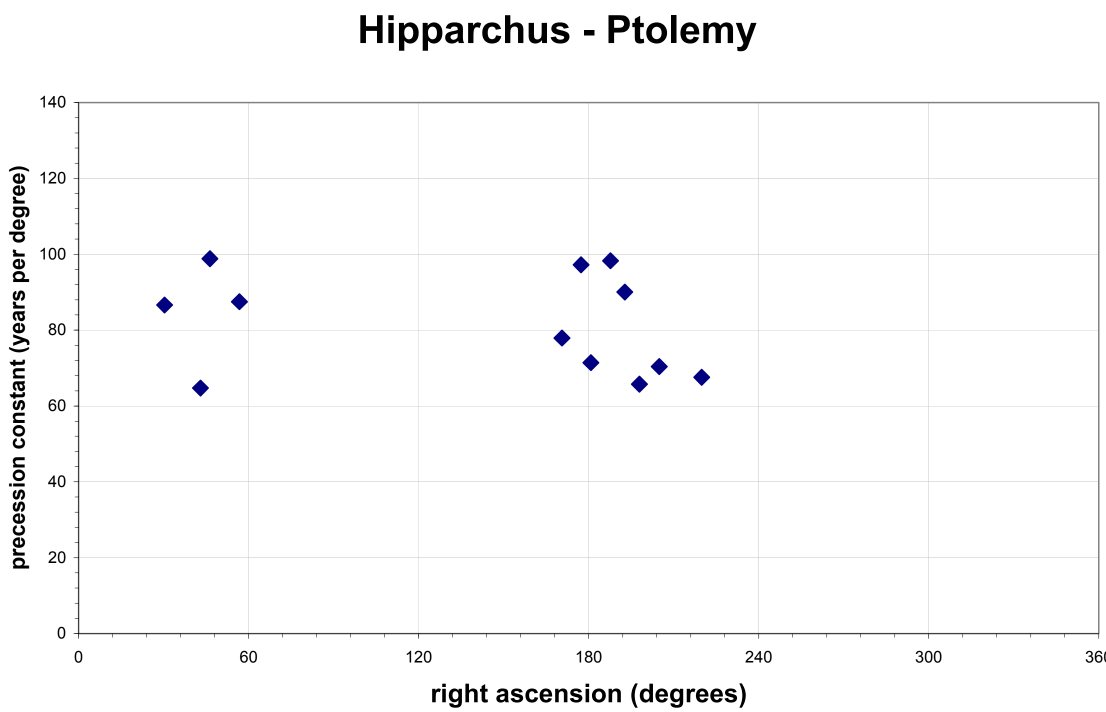

Figure 2. The precession constants for Ptolemy's stars. Four stars (α Ori, α Cma, α Gem, and α Leo) are so near a solstice that Ptolemy (or Hipparchus) might have decided to exclude them from consideration. The average value for the 12 stars shown is 81 years/degree.

References

[1] *Ptolemy's Almagest*, transl. by G. J. Toomer (London, 1984), p. 330 ff.

[2] J. B. J. Delambre, *Histoire de l'Astronomie Ancienne,* (1817, reprinted New York, 1965), v. 2, p. 252; H. Vogt, "Versuch einer Wiederstellung von Hipparchs Fixsternverzeichnis", *Astronomische Nachrichten*, 224 (1925), col 36; A. Pannekoek, "Ptolemy's Precession", *Vistas in Astronomy* 1 (1955) p. 73-96; R. R. Newton, "The Authenticity of Ptolemy's Eclipse and Star Data", *The Quarterly Journal of the Royal Astronomical Society,* 15 (1974) p. 107-121. A concise summary of all these is in G. Grasshoff, *The history of Ptolemy's star catalogue* (New York, 1990), p. 30-31, 59-61, 73-75, 81-83. It is somewhat curious that each of these commentators has ignored the position values Ptolemy provides in *Almagest* 7.3. For example, Pannekoek writes explicitly "He [Ptolemy]….states…that the same difference of declination is found for two points of the ecliptic situated *about the star's longitude* [italics added] at a mutual distance of 2°40′". Toomer, *op. cit.* (ref.1), p. 333, footnotes 62 and 63, at least expresses puzzlement over the discrepancies.

[3] Ptolemy, *Handbuch der Astronomie*, German tran. and annot. by K. Manitius. 2 vols., Leipzig (1912-13, reprinted 1963). See in particular vol. 2, p. 20-22 for a series of six footnotes in which Manitius computes the changes in declination of the ecliptic segments, assuming that the locations of the segments as given by Ptolemy are polar longitudes. When he can, Manitius further assumes the polar longitudes that Hipparchus gives in his *Commentary to Aratus*.

[4] D. Rawlins, "Hipparchan Precession-Math Spherical Trig Relics, Manitius' Discernment, Ancient Calculus?", *Proceedings of the XXth International Congress of History of Science*, ed. G. Simon and S. Debarat, Liege (20-26 July 1997), realized that the locations specified by Ptolemy might be right ascensions in the form customarily used by Hipparchus of sign and degree increment, and that the passage in *Almagest* 7.3 might be originally from Hipparchus.

[5] This assumption is therefore distinguished from the alternate assumption that Ptolemy himself chose the list of 12 stars.

[6] I am here using the date suggested by least-squares analysis of Aristyllos' declinations: see D. Rawlins, *Isis* (1982), p.263; *DIO* 1.2, p. 124, fn. 126; *DIO 4.1,* ‡3, fn. 40. All *DIO* issues may be conveniently found at *www.dioi.org*. See also Y. Maeyama, "Ancient Stellar Observations Timocharis, Aristyllos, Hipparchus, Ptolemy: the Dates and Accuracies", *Centaurus* 27 (1984) p. 280-310.

[7] Thus we are ignoring the fact that for one of the stars, Spica (α Virginis), Ptolemy tells us explicitly in *Almagest* 7.2 that Hipparchus knew, from the analysis of lunar eclipses, the ecliptical longitude of Spica in both Timocharis' time and his own. The values Ptolemy quotes, $\lambda = 172^\circ$ and $\lambda = 174^\circ$, are fairly accurate if we assume that the lunar eclipses in question were those that occurred on –283 Mar 17 and –134 Mar 21 (the true longitudes of Spica on those dates were 172.15° and 174.21°, respectively). If Hipparchus knew those dates, he could have estimated the rate of precession as about 74 years per degree, but Ptolemy didn't report such an estimate in the *Almagest* (by either Hipparchus or himself).

[8] Ptolemy tells us explicitly in *Almagest* VII.1 that Hipparchus had a globe. R. Nadal and J.-P. Brunet, "Le Commentaire d'Hipparque I. La sphère mobile", *Archive for history of exact sciences*, 29 (1984), 201-36 and "Le Commentaire d'Hipparque II. Position de 78 étoiles", *Archive for history of exact sciences*, 40 (1989), 305-54 concluded that Hipparchus plotted stars on his globe using right ascension and declination and used the globe to deduce the rising, setting, and transit times reported in the *Commentary*.

[9] D. Rawlins, *op. cit.* (ref. 4).

[10] Ptolemy is here comparing the changes in declination of the stars with the changes in declination of the corresponding ecliptic segments, but these are directly related to the estimated values of the precession constant, so I am mixing them intentionally in order to clarify the discussion.

[11] If, as Newton, *op. cit.* (ref. 2), alleged, Ptolemy fabricated the results for the six chosen stars, then it is unclear to me why he would have used the qualifications 'near' and 'approximate' to rather accurately characterize the level of agreement he found. In addition, D. Rawlins, "Ancient Geodesy: Achievement and Corruption", *Vistas in Astronomy*, 28 (1985) p. 257 shows that the accuracy of the remaining 12 declinations

that Ptolemy also claims as his own exceeds the error in geographical latitude that Ptolemy claims he used for his observations in Alexandria. Perhaps the simplest consistent scenario is that Ptolemy used 18 values for his time measured by someone who knew the latitude of Alexandria.

[12] O. Neugebauer, *A history of ancient mathematical astronomy*, (3 vols., Berlin, 1975), p. 280.

[13] G. Toomer, *op. cit.* (ref. 1), p. 330, fn. 56.

[14] Hipparchus, *Commentary on the Phenomena of Aratus and Eudoxus*, trans. Roger T. Macfarlane (private communication). Until this is published, the interested reader must use Hipparchus, *In Arati et Eudoxi phaenomena commentariorium*, ed. and transl. by K. Manitius (Leipzig, 1894), which has an edited Greek text and an accompanying German translation.

[15] D. Duke, "Hipparchus' Coordinate System", *Archive for history of exact sciences,* 56 (2002) 427-433.

[16] See, for example, O. Neugebauer, *op. cit.* (ref. 12), p. 277-80; G. J. Toomer, *Hipparchus*, Dictionary of Scientific Biography 15 (1978), p. 217; J. Evans, *The History and Practice of Ancient Astronomy*, (New York, 1998), p. 103; G. Grasshoff, "Normal star observations in late Babylonian astronomical diaries", *Ancient astronomy and Celestial Divination* (1999), ed. N. Swerdlow, p 127 and footnote 23.

[17] Toomer, *op. cit.* (ref. 1), p. 329.

[18] But see D. Rawlins, *DIO* 1.2 (1991), p. 127, which vigorously disputes Toomer's choice of 'computed' as a translation of the Greek word συναγομέναισ. For example, Ptolemy, *The Almagest,* trans. R. C. Taliaferro (1952) translates the same passage as "…we find nearly the same distances contained *as were recorded and brought forward by Hipparchus.*" It is also true that Manitius' German translation is more consistent with Taliaferro's version. However, a check of the on-line Liddell-Scott-Jones Greek Lexicon (which may be found at *http://perswww.kuleuven.ac.be/~p3481184/greekg/diction.htm*) clearly supports both translations, depending, of course, on the context. In the present context, Toomer's version appears to me favored.

[19] Toomer, *op. cit.* (ref. 1), in n. 110 on p. 344, n. 120 on p. 347, n. 31 and n. 34, 35 on p. 377. Toomer also includes another case in n. 117 on p. 346, but his discussion is in error in that footnote.

[20] Noel Swerdlow, private communication, 2001.

[21] J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler*, (New York, 1953), p. 203.