Expansion of Cold Plasmas from Higher Density to Lower Density

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Introduction

The Cold-Ion model is the limit of the Vlasov-Poisson equation as the electron temperature becomes negligible compared to the ion temperature. These equations are of particular interest because a shock forms as a Cold-Ion plasma of high density expands into a Cold-Ion plasma of lower density. In particular, in the 1D case, it was shown that the infinite spike forms as the solution of the Cold-Ion model becomes multi-valued, which is exhibited in the behavior of the Vlasov-Poisson equation with low electron temperature. It is shown in the asymptotic analysis of the linearized Cold-Ion model that an infinite spike propagates through the domain at $x \approx t$. [1]

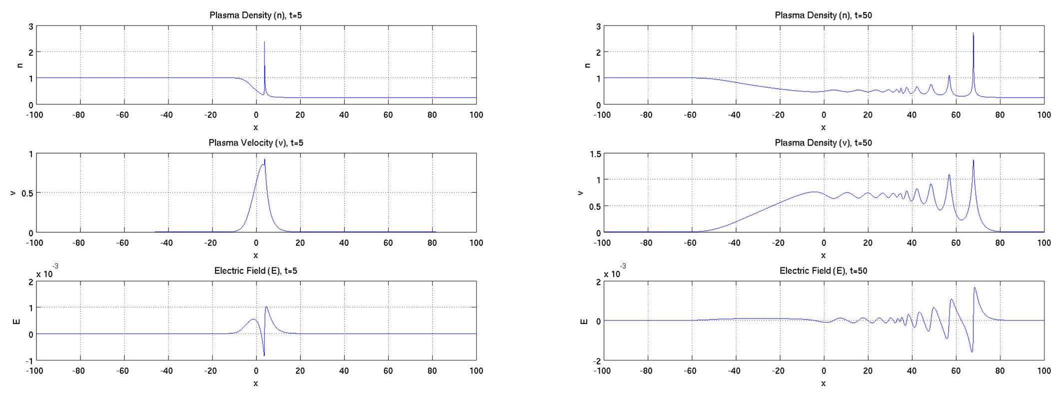

Figure 1: Plots depicting the expansion of high density cold plasma into lower density cold plasma in 1D

For the purpose of this research, the behavior of the shock in the expansion of cold-ion plasma for various cases are studied.

The Cold-Ion Model

The non-dimensional model is given below:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \tag{1}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\nabla \phi \tag{2}$$

$$\nabla^2 \phi = e^{\phi} - n \tag{3}$$

n denotes the plasma density, $\vec{v}$ denotes the plasma velocity, and ϕ denotes the electrostatic potential.

Initial Conditions

Let $\Omega = \Omega_1 \cup \Omega_2$. Ω_1 represents the portion of the domain where the high density plasma initially resides, Ω_2 represents the portion of the domain where the low density plasma initially resides. Ω represents the entire domain of the problem.
The initial condition is given as:

$$n(\vec{x}, 0) = \begin{cases} 1 \text{ if } \vec{x} \in \Omega_1 \\ n_r \text{ if } \vec{x} \in \Omega_2 \end{cases} \qquad \vec{v}(\vec{x}, 0) = \vec{0} \tag{4}$$

n_r denotes the plasma density ratio.

Computational Aspects

To model the expansion of cold plasma from high density to lower density, numerical methods were used to solve the partial differential equations. Newton's method was used with a second centered finite difference approximation was used to solve the nonlinear Poisson equation. The explicit Lax-Friedrichs scheme was chosen to solve the nonlinear hyperbolic portion of the system because of its dissipitive nature as well as its ability to preserve the monotonicity of the solution. Homogeneous Neumann boundary conditions were implemented to simulate an infinite domain.

The algorithm is computationally very expensive because Newton's method ideally must be used every time step of the simulation. However, it was found that by time-lagging the Poisson solve by a set number of time steps, the computational cost can be reduced. However, this method increases the truncation error of the numerical solution. The time-lag error incurred by this method is:

$$\epsilon = L \frac{\partial \nabla \phi}{\partial t} \Delta t + \mathcal{O}\left(\Delta t^2\right) \tag{5}$$

Where L is the positive integer time-lag parameter.

To reduce computation time, the multithreaded SuperLU package [2] was used to solve the matrix system in parallel.

2D Expansion in a Box

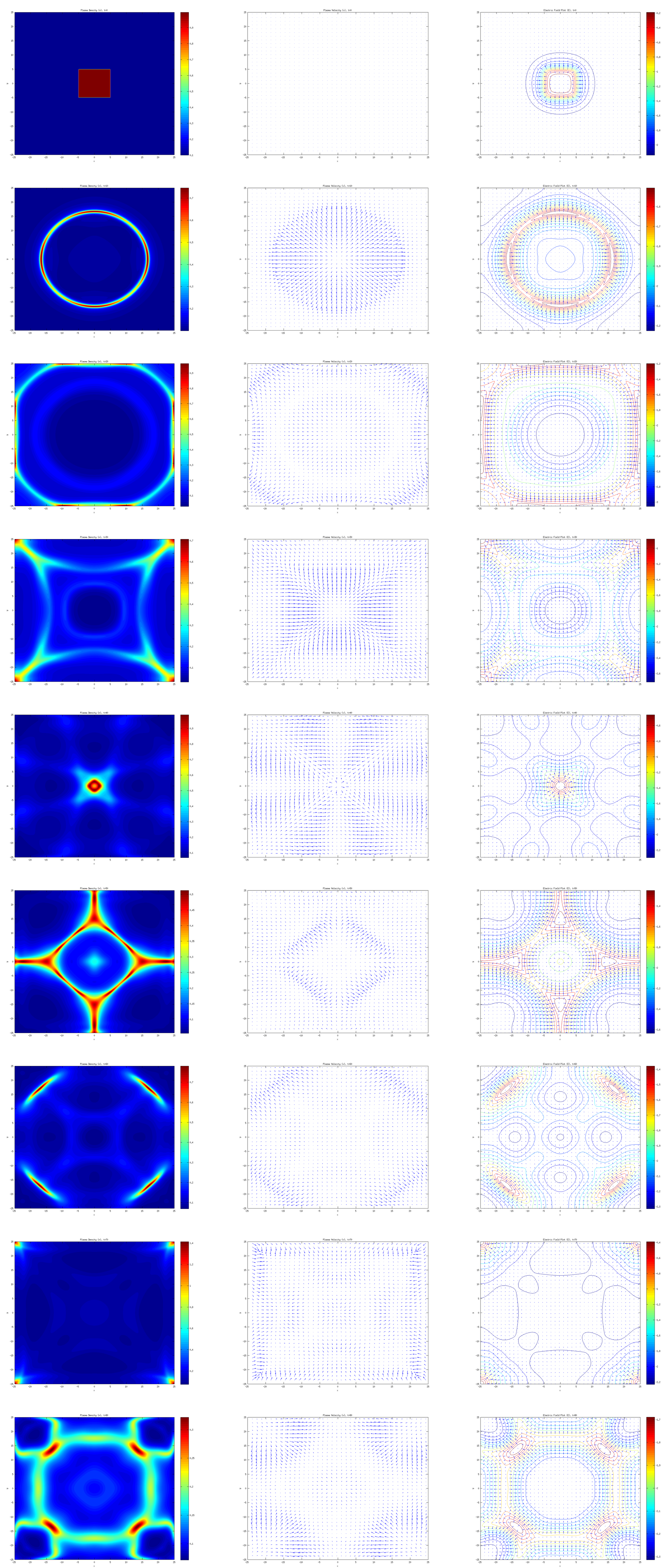

Figure 2: The expansion of a square region of plasma with high density into a plasma of low density from t=0 to t=80. Cold plasma density (left), velocity field (center), and electric field (right)

Conclusion

In this study, it can be concluded that in 2D, a spike forms as a higher density cold plasma expands into a cold plasma of lower density, similar to the 1D case. This is to be expected because the Cold-ion model is a generalization of the Euler equation coupled with the Poisson equation for the electrostatic potential of the fluid. However, the growth of the spike depends on the amount of high density plasma present in the initial condition.

Future Work

In the future, the bifurcations in the solutions of the Vlasov-Poisson equation with very low electron temperatures will be studied for various cases in one-dimensional flow.

References

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