1 Finite element method exercises

10. (10 points) For the case of piecewise linear trial functions on a uniform mesh of intervals with length h, show by direct calculation that the first stiffness matrix, κ_1, given in the lecture is

$$\int \phi_i' \phi_j' = \frac{1}{h} \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

and the second stiffness matrix, κ_2, is

$$\int \phi_j \phi_j = \frac{h}{6} \begin{bmatrix} 4 & 1 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & 1 & 2 \end{bmatrix}$$

11. (5 points) Complete the code for `class Mesh`. Be sure to test your code.

12. (5 points) Complete the code for `FunctionSpace.int_phi`. Be sure to test your code.

13. (10 points) Construct and run a program to solve the ODE

$$u'' + 2u' + u = f = (x + 2)$$

with Neumann boundary conditions. Its exact solution is

$$u = (1 + x)e^{1-x} + x(1 - e^{-x})$$