# Analysis of the Farnese Globe 

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A recent analysis ${ }^{1}$ of photographs of the Farnese globe by Schaefer concludes that ${ }^{2}$
The constellations on the Farnese Atlas are based on the now-lost star catalogue of Hipparchus. This is proved by

S1.1) the virtually perfect match with the constellation figures used by Hipparchus and only for these,
S1.2) the perfect match with the date of Hipparchus (with the exclusion of all other known candidate sources),
S1.3) the requirement that the source be a star catalogue such as that compiled by Hipparchus, and
S1.4) the many points of consistency with what we know about ancient Greek astronomy.

In order to determine whether any of these claims are valid, it will be useful to discuss in turn in the following:

1) what constitutes a proof of an Hipparchan origin, as opposed to, for example, a demonstration that the globe is consistent with an Hipparchan origin, but could also have other origins,
2) the importance of considering systematic errors when dating any star catalog of unknown origin,
3) comparisons between the globe and what is known of Hipparchus' data
4) possible other sources for the globe not considered by Schaefer.

The conclusion will be that nothing in Schaefer's paper constitutes a proof that Hipparchus is the only source of the data underlying the globe, that Schaefer's stated uncertainty in the date is much smaller than it should be, that there are many discrepancies between Hipparchus' known data and the globe, and that the possibility of sources other than the ones considered by Schaefer is potentially important to consider.

## I. How to Prove an Hipparchan Origin

Of the four points listed above, S1.4 - consistency with what we know of ancient Greek astronomy - clearly implies nothing more than that the globe is based on a reasonable approximation to the real sky, as opposed to the unrestrained imagination of some artist. Certainly no particular astronomer is implied.

Point S1.3, that the constellations appear to be based on some sort of catalog, must be considered along with Schaefer's additional conclusion that the typical errors of such a catalog are about $2^{\circ}$. Now the accuracy of Hipparchus' catalog is about $12^{\circ}{ }^{\circ},{ }^{3}$ and so knowing that the source catalog need be no more accurate than $2^{\circ}$ clearly does not help in
any way to establish Hipparchus as the source. Indeed, it is reasonable to imagine $2^{\circ}$ accuracy, which is about the same as four moon widths in the sky, as a level of accuracy achievable by just about any competent astronomer in Antiquity.

Point S1.2, that the date that best characterizes the configuration of the constellations on the globe is, with $65 \%$ probability, somewhere between 180 B.C. and 70 B.C., and with $95 \%$ probability anywhere between 235 B.C. and 15 B.C., is clearly consistent with Hipparchus but hardly requires Hipparchus as the source.

The final point to consider is S1.1, the claim of a 'virtually perfect match' with the constellation figures used by Hipparchus. Even if the match is indeed virtually perfect, which it is not, then we would conclude at most that the globe is consistent with an Hipparchan origin, but it most certainly does not isolate Hipparchus as the only source.

So clearly it is one thing to conclude that what we see on the globe is consistent with an Hipparchan origin. To reach such a conclusion it is indeed adequate to establish the four points S1.1-4, since those points, if they are true, are sufficient to show that we cannot rule out Hipparchus as the source.

But to conclude that the source is proved to be Hipparchus, it is necessary to provide substantial positive evidence uniquely linking Hipparchus to the depictions on the globe. As we will discuss below, such positive evidence is demonstrably lacking in Schaefer's paper. Furthermore, it will be shown that in spite of the claim of a 'virtually perfect match', there are so many serious discrepancies between Hipparchus' Commentary to Aratus and Eudoxus and the globe that even the consistency is highly doubtful. Then two alternatives are suggested:

1) that the globe is modeled on information originating from some source that we do not know. Given the small fraction of ancient documents that have come down to us, such an option should of course be considered not only plausible, but even likely.
2) that the useful information content on the globe is so distorted by the artist(s) that no reliable conclusion can be reached, including any resolution of the idea that Hipparchus is or is not the source.

## II. Systematic Errors in Dating a Star Catalog

Schaefer's estimated error of $\pm 55$ yrs on the date is the spread due to the random errors in the data, and can be made arbitrarily small by including more star coordinates in the determination. Unfortunately, that error does not include the effect of any mistake (or systematic error) in the position of the ancient astronomer's equinox. The neglect of this second source of error is easily illustrated by an example: the star catalog of the Almagest. The average error for each star in the catalog is about $12^{\circ}$, but the position of the equinox is off by about $1^{\circ}$. A chi-square analysis of the catalog gives a mean date of
about A.D. $53.8 \pm 1.5 \mathrm{y}$. The uncertainty in this example is so small because we have over 1,000 stars in the data sample. But since Ptolemy's stated epoch of the Almagest star catalog is A.D. 137, this is wrong by some 83 years, or more dramatically, by about 55 standard deviations!

In addition, if the input data for our analysis comes not from a catalog, but from a globe, then there are additional problems. Based only on evidence from the globe itself, the equinox could easily be in error by as much as Schaefer's estimate of the overall accuracy of the underlying catalog, some $2^{\circ}$ or so, which translates to an uncertainty in the date of about $\pm 144$ yrs. Since we must allow for additional possible errors in the artist's transfer of the data to the globe, this is a conservative estimate. Adding the statistical scatter in star positions, a minimum uncertainty of $\pm 200$ years must be expected. There is, therefore, no basis to the claim of a 'perfect match' with the date of Hipparchus.

## III. Comparing the Globe with Hipparchus' Known Data

The evidentiary basis behind S1.1 is the information in the following three lists. Schaefer first finds that there are nine points of disagreement between the globe and all four sources he considers, i.e. Aratus and Eudoxus, Hipparchus, the Catasterismi of Eratosthenes, and the Almagest star catalog of Ptolemy:

S2.1) On the globe the horn of the Bull does not touch the foot of the Charioteer
S2.2) the head of Andromeda does not overlay the navel of Pegasus
S2.3) the rectangular feature above Cancer is not mentioned by any source
S2.4) Sagitta is missing
S2.5) Triangulum is missing
S2.6) Ursa Minor is missing
S2.7) the ecliptic crosses the equator about $5^{\circ}$ west of the colures.
S2.8) no string is attached to the northern fish of Pisces
S2.9) Sagittarius is missing a cloak-strap
Second, Schaefer finds just one potential difference between Hipparchus and the globe:
S3.1) on the globe the head of the following twin (Pollux) is clearly on the tropic, whereas Hipparchus says it is $6^{\circ}$ north of the tropic

This single discrepancy is dismissed on the grounds that a single $1.7 \sigma$ error is not sufficient to reject an Hipparchan origin. Schaefer also considers a second potential difference, that the Balance is depicted on the globe while Hipparchus usually refers to the Claws, but this difference is dismissed on the grounds that the Balance is used in C3.1.5. ${ }^{4}$

Schaefer then points out twelve items where Hipparchus' descriptions agree with the globe:

S4.1) the human part of the Centaur is under Virgo
S4.2) the feet of the Charioteer are north of the tropic

S4.3) all of Perseus is north of the tropic
S4.4) only the beak of Cygnus is touching the tropic
S4.5) the top of Ophiuchus' head is on the tropic
S4.6) the knees of Ophiuchus are substantially south of the equator
S4.7) the neck of Cepheus is on the Arctic circle
S4.8) the head of Draco is only close to the Arctic circle
S4.9) Crater is south of the equator
S4.10) Corvus is south of the equator
S4.11) the declination of the Arctic circle is $53^{\circ} \mathrm{N}$.
S4.12) Eridanus has a second westward segment
Before considering the above items in detail, it is useful to also consider the following list of items, all of which are clear discrepancies between Hipparchus and the globe. ${ }^{5}$ In many cases the discrepancies are visually apparent on the globe and require no measurement. In other cases it is helpful to compare the coordinates of features on the globe with coordinates provided by Hipparchus, and to do this I have measured the coordinates using the Folkes map of the globe. The details of the method and the accuracy of the map, which is generally good but far from perfect, are discussed in the Appendix. It is important to stress, however, that while the use of numerical coordinates deduced from the Folkes map is useful and even interesting in some cases, not a single conclusion of this paper requires the use of the Folkes map. Indeed, all the conclusions follow from the simple logic of what does and does not constitute a proof of Hipparchan origin, the impact of systematic errors in establishing the uncertainty in the date of the globe, and the many discrepancies with Hipparchus that are visually apparent and that can be established by simply looking at photographs of the globe from various sources.

The first five discrepancies are related to iconographic features present or absent on the globe:

1) The southern Crown is on the globe, and was known to Geminus and Ptolemy. It is the only southern constellation on the globe that is not in Hipparchus' list of rising and setting southern constellations.
2) On the globe Ara is shown right side up and tilted, instead of inverted, as Hipparchus (and also Ptolemy in the Almagest) describes it. This inversion is clearly established by Hipparchus' language that, e.g. the lip of the Altar is the first star to set while the last to set is the northern of those in the base (C3.2.6). The contrast with Aratus, who agrees with the globe, is discussed in detail by Kidd. ${ }^{6}$
3) The globe shows the Balance supported by one of the Claws of the Scorpion. The Claws are mentioned 92 times by Hipparchus, but the Balance is mentioned in the Commentary just once. Manitius comments, no doubt correctly, that the appearance of the word Balance is a scribal slip. The oldest surviving manuscript of the Commentary dates to about A.D. 1100, so such a slip is far from unlikely.
4) The globe shows wings on Pegasus, a feature never mentioned by Hipparchus. Schaefer uses the fact that the wings are explicitly denied in the Catasterismi to rule out Eratosthenes, but says this does not apply to Hipparchus since he never explicitly denies the wings. However, Hipparchus is very explicit about the rising and setting of Pegasus, and says that the bright star in the hip is the last star of Pegasus to rise and set. It is absolutely clear that if Pegasus was thought by Hipparchus to have wings as drawn on the globe, then this could not be true. We may then very safely conclude that Hipparchus did not have wings on Pegasus, and so this is a significant discrepancy with the globe.
5) The globe shows a rectangular figure just north of the Crab that matches nothing in Hipparchus. The fact that it also matches nothing in Aratus, Eratosthenes, or Ptolemy could mean that it is an astronomically meaningless addition by some artist, but it could just as easily be used as evidence for some unknown source. Several scholars have suggested that the figure represents the throne of Caesar mentioned by Pliny. ${ }^{7}$

The next thirteen discrepancies are generally large, and many involve figures misaligned with respect to some circle on the globe:
6) "...the following head of the Twins is further north of the equator by $30^{\circ} \ldots$ " (C1.10.1). On the globe the head of the following twin is nicely centered between the declination lines of $20^{\circ}$ and $30^{\circ}$, with a width of perhaps $3^{\circ}$. This point is mentioned by Schaefer.
7) "For Perseus lies in such a position that the portions toward its head are toward the north, and the feet are toward the south, with the head slightly tilted to the east." (C1.10.5). On the globe the body of Perseus is drawn significantly more east-west than north-south, as Hipparchus describes it.
8) "Likewise, both the right hand and head of Perseus lie further east than the circle under discussion [the equinoctial colure] by about one third of a zodiacal sign [i.e. $10^{\circ}$ ]." (C1.11.17). On the globe the right hand and head of Perseus touch the $0^{\circ}$ colure, and the wrist and chin do not exceed $4^{\circ}$ right ascension.
9) "For the stars in her [Andromeda's] right shoulder are further north than the tropic." (C1.10.6). On the globe, Andromeda’s right shoulder is clearly south of the tropic.
10) "...the right shoulder [of Ophiuchus] is north of the equator by nearly $7^{\circ}$, the left nearly at $15^{\circ}$. ." (C1.10.7). On the globe, the right shoulder has a declination of about $16^{\circ}$, the left $20^{\circ}$. Hence Ophiuchus is depicted on the globe more upright than Hipparchus describes.
11) Hipparchus quotes Aratus as writing "The Maiden goes a little farther south and does not touch it [the summer tropic]..." (C1.10.7) and then Hipparchus comments, in C1.10.11, "He seems to me to have said these points in agreement with celestial phenomena." On the globe, the right hand of Virgo clearly touches the tropic.
12) "And of the head of the Water-snake, the leading stars have positions on a meridian at $10^{\circ}$ of the Crab and more." (C1.11.11) ${ }^{8}$. On the globe the head of Hydra lies entirely beyond $105^{\circ}$, i.e. $15^{\circ}$ of the Crab in Hipparchus’ terminology.
13) "Cepheus' left hand, moreover, diverges very far to the east of the circle under discussion [at right ascension $270^{\circ}$ ]. For even the stars which are most leading in its head hold positions more than $10^{\circ}$ of the Water-pourer, and the bright star in the left hand, which certain people place in the shoulder, holds a position at $25^{\circ}$ of the Water-pourer." (C1.11.11). On the globe there is an elbow and not a hand at right ascension $325^{\circ}$.
14) "Of the stars in the Ram, however, the one which is northern and lies upon the nose is a little less than $78^{\circ}$ away [from the pole]; similar to it is the northern of the stars in the tail." (C1.6.8) Hence Hipparchus is saying that the nose and tail of Aries are at declination $12^{\circ}$. On the globe, they are noticeably further north, at about 17-18 ${ }^{\circ}$.
15) "For the entire [Ram] is north of the equator; but only the star in its rear feet is borne upon the equator itself." (C1.10.18) On the globe the front feet are distinctly below the equator.
16) "For Arcturus is $59^{\circ}$ from the north pole..." (C1.8.16). So Hipparchus says the declination of Arcturus is $31^{\circ}$. Aratus puts Arcturus 'below the belt' of Bootes, Eratosthenes puts it 'between the knees', Hyginus puts in 'on the belt', and Ptolemy puts it 'between the thighs'. On the globe $31^{\circ}$ is about midcalf on Bootes, so Hipparchus' position is well to the south of any ancient placement before or after Hipparchus’ time.
17) "...[for Centaurus] the southern of the stars in the rear legs is at $13^{\circ}$ of the Maiden." (C1.8.21). So Hipparchus says the southern of the stars in the rear legs has a right ascension of $163^{\circ}$. On the globe the rear leg which is the most southern has a right ascension of about $153^{\circ}$, well short of Hipparchus’ $163^{\circ}$.
18) "the star in the right shoulder [of Centaurus] is at $7^{\circ}$ of the Claws [ $187^{\circ}$ ] (C1.8.21)....The star in the right hand of the Centaur lies east of the circle under discussion [at $180^{\circ}$ ] by about one fourth of a zodiacal sign; for it holds a
position at $8^{\circ}$ of the Claws [188${ }^{\circ}$." (C1.11.17). On the globe the right hand of the Centaur is extended far more than $1^{\circ}$ from the right shoulder.

The next seven discrepancies follow from Hipparchus' rising and setting data, which more or less directly establish the size and position of constellations, in conflict with what is seen on the globe:
19) The rising and setting of $\mathrm{CrB}(\mathrm{C} 2.5 .2$ and C 2.6 .2 ) and the transits of stars in CrB (C2.5.9 and C2.5.10) establish that the constellation extends about $9^{\circ}$ in right ascension (about $210^{\circ}$ to $219^{\circ}$ ) and about $6^{\circ}$ in declination (about $35^{\circ}$ to $41^{\circ}$ ). On the globe CrB extends about $15^{\circ}$ in right ascension (from about $215^{\circ}$ to $230^{\circ}$ ) and about $10^{\circ}$ in declination (about $38^{\circ}$ to $48^{\circ}$ ).
20) Similarly, for Hipparchus (C2.5.4 and C2.6.4) Ophiuchus extends about $36^{\circ}$ in declination (about $-20^{\circ}$ to $16^{\circ}$ ). On the globe Ophiuchus extends about $46^{\circ}$ in declination (about $-22^{\circ}$ to $24^{\circ}$ ).
21) Similarly, for Hipparchus (C2.3.9) the nose of the southern fish of Pisces is at about $318.6^{\circ}$. On the globe the nose of the southern fish is at about $314^{\circ}$.
22) Similarly, for Hipparchus (C2.5.14 and C2.6.14) Delphinus extends northward to about declination $10.5^{\circ}$. On the globe Delphinus extends northward to about $20^{\circ}$ declination.
23) Similarly, for Hipparchus (C2.5.6 and C2.6.6) Lyra extends from about $31^{\circ}$ to $39^{\circ}$ in declination. On the globe Lyra extends from about $26^{\circ}$ to about $44^{\circ}$. We can be less sure about the extension in right ascension, but it appears that Hipparchus would take it as about $9^{\circ}$, while on the globe it is about $12^{\circ}$.
24) Similarly, for Hipparchus (C2.5.13 and C2.6.13) the Eagle extends from about $3^{\circ}$ to $8^{\circ}$ in declination. On the globe the Eagle extends from about $8^{\circ}$ to $24^{\circ}$ in declination.
25) Similarly, for Hipparchus (C2.6.11) the bright star in the Horse's mouth has a declination of about $2^{\circ}$ north, while on the globe the Horse's mouth lies at about $10^{\circ}$ north.

At the end of the Commentary Hipparchus gives a list of stars marking each of the 24 hours of right ascension. Among these, about 40 can be plausibly identified on the globe. Most are largely correct, both on the globe and in the sky, and so are consistent with an Hipparchan origin, but do not require it. The following ten items, though, are substantially at variance with Hipparchus' data:
26) the star at the base of the Water-snake's neck ( $\zeta$ Hya) has $\alpha=105^{\circ}$, but on the globe the right ascension is about $111^{\circ}$.
27) the star in the right hand of Ophiuchus, which also lies in the body of the snake ( $\tau$ Oph), has $\alpha=210^{\circ}$, but on the globe the right ascension is about $215^{\circ}$.
28) the star that precedes the bright star in the Crown ( $\beta \mathrm{CrB}$ ), has $\alpha=210^{\circ}$, but on the globe the right ascension is about $215^{\circ}$.
29) the star in the middle of Hercules' right knee ( $\varphi$ Her) has $\alpha=22534^{\circ}$, but on the globe the right ascension is about $235^{\circ}$.
30) the following star of the three in the Archer's head ( $\pi \mathrm{Sgr}$ ) has $\alpha=255^{1} 1_{2}{ }^{\circ}$, but on the globe the right ascension is about $250^{\circ}$.
31) the middle star in Cepheus' head ( $\zeta$ Сер) has $\alpha=3143^{3}{ }^{\circ}$, but on the globe the right ascension is about $310^{\circ}$.
32) the star in the middle of Cassiopeia's body ( $\eta$ Cas) has $\alpha=344 \frac{1}{4^{\circ}}$, but on the globe the right ascension is about $330^{\circ}$.
33) the most bright star of those in the Gorgon’s head ( $\beta$ Per) has $\alpha=151^{\circ}{ }^{\circ}$, but on the globe the right ascension is about $23^{\circ}$.
34) The $4^{\text {th }}$ and $7^{\text {th }}$ stars in Orion's pelt ( $\pi_{1}$ and $\pi_{4}$ Per) have $\alpha=45^{\circ}$, but on the globe the right ascension is about $52^{\circ}$.
35) the middle of the three bright stars in the twins' knees ( $\zeta \mathrm{Gem}$ ) has $\alpha=74 \frac{1}{4} 4^{\circ}$, but the globe has the right ascension at about $65^{\circ}$.

One way to establish a direct link between Hipparchus and the globe, if such a link exists, would be to check whether Hipparchus' known large errors in star coordinates are reflected on the globe. It was, after all, the coincidence of these large errors (and a huge number of medium size errors, for that matter) on the same stars that provided the crucial evidence linking Hipparchus and the Almagest star catalog. ${ }^{9}$ These errors include the misplacements of the left hand of the leading Twin ( $\theta$ Gem) , the right front leg of Centaurus ( $\alpha$ Cen), the double star at the edge of the Altar ( $\beta / \gamma$ Ara), and the star at the tip of the tail of Hydra ( $\pi$ Hya). Only one such large Hipparchan error is, in fact, clearly reflected on the globe. Hipparchus puts $\pi$ Hya at declination -20.5 (C3.1.1b and C3.2.1), but it should be at about $-15.3^{\circ}$. On the globe the tip of the tail of Hydra is at about $-20^{\circ}$. But in the face of all the other evidence listed, this is far from sufficient to warrant a conclusion that Hipparchus is the source of the star coordinates on the globe.

The items above clearly establish that Schaefer's list of just one instance where Hipparchus is not in agreement with the globe is much too short. Instead, we must conclude that Hipparchus is either not the source or, at the very least, not the only source.

Finally, considering Schaefer's list of twelve items (S4.1 - S4.12) that support an Hipparchan origin, we may note that S4.2, S4.3, S4.4, and S4.9 are wrong. Simply looking at the globe shows that regarding S4.2, one foot of the Charioteer is well below the tropic. Regarding S4.3, Perseus' foot is clearly touching the tropic. Regarding S4.4, the beak of Cygnus does not touch the tropic, although it is close. And regarding S4.9, the handle of Crater is clearly not south of the equator. Beyond that, while the rest of the items are true, these items certainly do not suggest that Hipparchus alone must be the source of the data.

## IV. Possible Alternate Sources for the Globe

Going back now to consider Schaefer's second list (S2.1 - S2.9), he concluded that since all nine points differ from each of the four sources he considered, there was no useful information included. Under the circumstances as we now understand them, however, it could just as plausibly be concluded that those nine points contribute evidence that the source is simply not one of Aratos, Eratosthenes, Hipparchus, or Ptolemy.

In particular, S2.7, the misalignment of the colure with the equinoctial point, is strongly at variance with an Hipparchan origin. While it is easy and expedient to blame the problem on the artist(s), such reasoning can be continued indefinitely whenever we find any sort of discrepancy, and so we are able to learn nothing (which could, of course, be the correct conclusion: that the artists have effectively washed out all useful information from the globe). It is also interesting that the Mainz globe shows the very same offset of the colure and the intersection of the equator and ecliptic, although the size of the offset is a bit smaller. ${ }^{10}$ One explanation, suggested by both the original investigations of Bianchini and Cassini in older times and Valerio in 1989, is that the offset is simply an attempt to model or depict precession. ${ }^{11}$ Perhaps related to this are four short line segments perpendicular to the ecliptic on the globe, and that connect the ecliptic with its parallel boundary markers offset some $612^{\circ}$. These line segments - at the feet of the Twins, at the shoulder of the Virgin, at the autumnal colure, and by the right foot of Ophiuchus - are all very close to multiples of $30^{\circ}$ apart. This suggests, therefore, that the lines are marking zodiacal segments which are slightly offset from the point where the equator and the ecliptic cross. It is not impossible that such a configuration could have been considered at a time when precession was not fully understood. This is all reminiscent of Ptolemy's mention in Almagest 7.3 that at one point Hipparchus considered the option that precession affected only the zodiacal stars, but not those in the northern and southern constellations. ${ }^{12}$

An example of a possible source not considered by Schaefer is the Aratus ascribed to Germanicus Caesar, ${ }^{13}$ written most likely in the years between 4-14 A.D. ${ }^{14}$ Some features in Germanicus' poem are interesting because they are clearly seen on the globe, and are inconsistent with Hipparchus as a source:

1. the Southern Crown is explicitly listed among the southern constellations, but not mentioned by Hipparchus.
2. the wings of Pegasus are explicitly mentioned several times in Germanicus, but are not mentioned by Hipparchus (see item (4) in the list of 42 items above).
3. Not only Andromeda, but also Cepheus and Cassiopeia are described as dramatically stretching out their hands or arms, just as all three are shown on the globe. That Hipparchus does not allow for any significant arm length in either Cepheus or Cassiopeia is clearly established by the rising and setting statements he gives in C2.5.8, C2.5.9, C2.6.8, and C2.6.9.

Since each of these items is common to Germanicus and the globe, but not to the sky, they are examples of positive links between Germanicus and the globe. It is doubtful, however, whether they are conclusive, since it cannot be ruled out that all of them were considered reliable folklore items at the time, and so Germanicus could quite easily be one of many possible, and substantially equivalent, sources for the artist(s) who created the globe. In addition, there are many instances of discrepancies between Germanicus’ descriptions and the globe, so Germanicus could hardly be the sole source for the artist. Still, the influence in part of either Germanicus or some equivalent source can hardly be excluded.

## Summary

Schaefer certainly deserves credit for undertaking a new investigation of the Farnese globe, but in contrast to what he claims, his work does not prove that Hipparchus is the source of the constellation data on the Farnese globe. First, the true uncertainty of the date of the astronomical data on the globe is at least $\pm 200$ yrs. Second a convincing proof would have to provide clear positive links between what we know about Hipparchus' data and what we see on the globe, and that evidence is not in the paper. In fact, there are many points of disagreement between the Commentary and the globe, as well as many points of agreement. Even if Hipparchus is not the source, if the real source is basing his data on the sky, and if that source is reasonably competent (at the $2^{\circ}$ level), then of course we would expect many points of agreement, and these points would indicate no particular source. But the points of disagreement between Hipparchus' data and the globe are more than enough to establish that it is highly unlikely that a proof of Hipparchus as source can ever be convincing.

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## Appendix

The first serious examination of the Farnese globe in modern times was done by Valerio, a professor of mathematics and an expert in cartography. ${ }^{15}$ By direct physical examination and measurement he established that the circumference of the globe is about 2030 mm , so that one degree of a great circle is about 5.6 mm . The tropics and polar circles are not a constant distance from the equator. For example, distance from the equator to the southern tropic varies between $138-146 \mathrm{~mm}$, thus about $\pm 1^{\circ}$, and the distance to the northern polar circle varies between $314-329 \mathrm{~mm}$, thus about $\pm 1 ; 20^{\circ}$. The mean declinations established by Valerio put the northern tropic at $25 ; 30^{\circ}$, the southern tropic at $-25 ; 06^{\circ}$, the northern polar circle at $56 ; 43^{\circ}$, and the southern polar circle at $55 ; 26^{\circ}$. The mean obliquity of the ecliptic is then about $25 ; 18^{\circ}$. The mean width of the zodiacal borders is $13 ; 29^{\circ}$. Valerio emphasizes that all the measures are "taken with a grain of salt" and that errors of a few millimeters are to be considered.

Valerio also pointed out that among the earlier investigations of the globe there is an excellent map by Folkes. Moreover, Valerio demonstrated that Folkes had made a true stereographic projection (see the Figure). Folkes was working from a plaster cast of the original globe, and so would have been able to make hundreds of point-to-point angular measurements. ${ }^{16}$ For the convenience of the reader, gridlines in right ascension and declination have been added to the maps at $10^{\circ}$ spacing in each coordinate. ${ }^{17}$ The original map was included in Richard Bentley's 1739 edition of the Astronomicon of Manilius, and it is easy to establish, as follows, that the accuracy of his map is at least as good as Schaefer's photogrammetric results.

The right ascension $\alpha$ and declination $\delta$ of a given point on the map may be recovered as follows. ${ }^{18}$ Let $x$ and $y$ be the cartesian coordinates of the desired point, and let the center of the map correspond to the spherical coordinates $\alpha_{0}$ and $\delta_{0}$. Compute $\rho=\sqrt{x^{2}+y^{2}}$ and $c=2 \arctan (\rho / 2 R)$, where $R$ is the radius of the globe (and hence, by definition, half the radius of the map). Then the stereographic projection gives

$$
\delta=\arcsin \left(\cos c \sin \delta_{0}+y \sin c \cos \delta_{0} / \rho\right)
$$

and

$$
\lambda=\lambda_{0}+\arctan \left(\frac{x \sin c}{\rho \cos \delta_{0} \cos c-y \sin \delta_{0} \sin c}\right) .
$$

Using standard image processing software and a digital scan of the map it is straightforward to measure, on a pixel by pixel basis, the size of the map, the position of the center of the map, and the $x$ - $y$ coordinates of any point on the map. Following Schaefer, I measure coordinates with respect to the equator and the colures, although the intention of the artist or the original astronomer is, at the very least, ambiguous.

Sampling 5-7 points on each of the major coordinate lines drawn on the map gives the following results: the northern and southern polar circles are drawn at about $54.9^{\circ}$ and $55.1^{\circ}$. The northern and southern tropics are drawn at about $23.5^{\circ}$ and $-26^{\circ}$. The vernal colure is offset about $6.1^{\circ}$ and the autumnal colure about $5.3^{\circ}$ from their equinoctial points. All of these results are in good agreement, within the estimated uncertainties, of both Valerio's measurements on the globe and his measurements of the Folkes map, and of Schaefer's photogrammetric analysis of the globe.

Schaefer gives some 47 globe features which lie on or very near major circles on the globe. Each of these features is accurately reproduced on the map. In addition, I have measured on the map each of the 23 points on the globe specified by Schaefer in Table 5 of his paper, and my results are compared to his in the table. There are disagreements in the results for the muzzle of Aries (which seems to be due to a slip in Schaefer's calculations) and for the declination of Andromeda's head (which could be an error in Folkes' map). Neglecting these two cases, my results and Schaefer's are in excellent agreement: the average absolute discrepancy is about $1.7^{\circ}$ in right ascension and $2.6^{\circ}$ in declination. Both are consistent with our estimated measurement uncertainties. Finally, there are five good pictures of the globe in the Encyclopedia of World Art, ${ }^{19}$ and another eight photographs in Thiele. ${ }^{20}$ Close comparison of available photographs and the Folkes map does reveal several additional instances of distortions on the map, and so there is no reason to regard the map as a particularly high quality rendition of the globe.



## Figure caption:

The celestial sphere of the Farnese globe mapped in a true stereographic projection by Martin Folkes. The engraving was appended to the English edition of Manilio's
Astronomicon edited by Richard Bentley in 1739. The grid lines are a modern addition by Daan Strebe, the author of Geocart (see http://www.mapthematics.com).

## REFERENCES

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${ }^{2}$ B. E. Schaefer, ibid., p. 182, item (7).
${ }^{3}$ Dennis W. Duke, "Dating the Almagest Star Catalogue Using Proper Motions: A Reconsideration," Journal for the History of Astronomy, xxxiii (2002) 45-55.
${ }^{4}$ the notation Cx.y.z refers to standard section in Hipparchus' Commentary on the Phenomena of Aratus and Eudoxus. I am using the English translation of Roger MacFarlane (private communication) with the assistance of Paul Mills. Until this is published, the interested reader must use Hipparchus, In Arati et Eudoxi phaenomena commentariorium, ed. and transl. by K. Manitius (Leipzig, 1894), which has an edited Greek text and an accompanying German translation.
${ }^{5}$ I thank D. Rawlins (private communication) for pointing out items 4, 7, 15, 23, and the miscomputed negative right ascension of Aries’ muzzle noted in the Appendix.
${ }^{6}$ D. Kidd, Aratus Phenomena, (Cambridge, 1997) 326.
${ }^{7}$ E. Künzl, Ein römischer Himmelsglobus der mittleren Kaiserzeit. Jahrbuch des Römisches-Germanischen Zentralmuseums Mainz 47, Jahrgang 2000 (p. 535), and references therein; E. Künzl, "The globe in the Roemisch-Germanische Zentralmuseum Mainz: The only complete celestial globe found to-date from classical Greco-Roman antiquity", Der Globusfreund, Nr. 45/46 (1998), 81-154.
${ }^{8}$ In this and many other cases Hipparchus specifies positions on circles of constant declination by dividing the circle into twelve $30^{\circ}$ segments and referring to each section with the corresponding name of the zodiacal segment. For a full discussion see D. W. Duke, "Hipparchus’ Coordinate System", Archive for History of Exact Sciences, 56 (2002) 427-433.
${ }^{9}$ G. Grasshoff, The history of Ptolemy's star catalogue (New York, 1990); D. W. Duke, "Associations between the ancient star catalogues", Archive for history of the exact sciences 56 (2002) 435-450; D. W. Duke, "The Depth of Association Between the Ancient Star Catalogues", Journal for the History of Astronomy xxxiv, (2003) 227-230. ${ }^{10}$ E. Künzl, ibid. (ref. 6).
${ }^{11}$ V. Valerio, "Historiographic and numerical notes on the Atlante Farnese and its celestial sphere", Der Globusfreund, xxxv/xxxvii (1987), 97-124 is an excellent account of the history of astronomical investigations of the Farnese globe. The paper is available online at http://www.coronelli.org/farnese/. More recent details are available at V. Valerio, L'Atlante Farnese e la rappresentazione delle costellazioni, in Eureka, il genio degli antichi, a cura di Eugenio Lo Sardo, Napoli, Electa Napoli 2005, pp. 233-239. See also S. De Meis, "Astronomical reflexes in ancient coins", Commerce and Monetary Systems in the Ancient World: Means of Transmission and Cultural Interaction, Proceedings of the Fifth Annual Sympoium of the Assyrian and Babylonian Intellectual Heritage Project, Innsbruck, Austria, October $3^{\text {rd }}-8^{\text {th }} 2002$.
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${ }^{16}$ G. Thiele, Antike Himmelsbilder (Berlin, 1898), 27-45. An English translation by Hugh Thurston of Thiele's discussion of the Farnese globe is available at www.csit.fsu.edu/~dduke/thiele.pdf.
${ }^{17}$ the gridlines were added by Daan Strebe, the author of Geocart (see http://www.mapthematics.com).
${ }^{18}$ see, e.g., http://mathworld.wolfram.com/StereographicProjection.html.
${ }^{19}$ H. Stern, "Classical Antiquity" in the entry titled "Astronomy and astrology" in
Encyclopedia of world art (New York, 1960), ii, cols 48-57 and plate 23.
${ }^{20}$ G. Thiele, ibid. (ref. 16).

